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Size and Shape of the Human First Permanent Molar: A Fourier Analysis of the Occlusal and Equatorial Outlines

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ABSTRACT Form can be viewed as a combination of size and shape. Shape refers to the boundary outline independently from its orientation, relation to reference planes, and dimension (or size). Shape and its changes could be quantified by mathematical methods such as the Fourier series. In this investigation, Fourier analysis has been used to quantify the morphologic characteristics (size and shape) of the outline of the occlusal surface and maximum circumference (equator) in 259 normal, healthy human first permanent maxillary and mandibular molars and to assess the effect of sex. Large within-group variability was found in the Fourier coefficients. Both equatorial and occlusal molar areas were on average larger in male than in female homologous teeth, but the difference was statistically significant only for the equatorial areas. The mean ratios between equatorial and occlusal dental areas were independent from arch (maxillary and mandibular), side, or sex. Both equatorial and occlusal outlines of left and right homologous molars within sex and arch were similar, without size and shape differences. Similarly, no sex differences in shape were found in the comparison of homologous teeth. The method used in the present study could supply information about dental shape in both its entirety and local variations. In particular, the method is extremely sensitive to local variations in dental shape, and it could be usefully employed to compare single teeth to a standard. Am J Phys Anthropol 108:281–294, 1999.

The form of the human teeth and their normal variations as well as the influence of sex and race have been widely analyzed by both anatomical and anthropological descriptions and clinical investigations (Abe et al., 1996; Ash, 1993; Dalidjan et al., 1995; DuBrul, 1986; Hattab et al., 1996; Kitagawa et al., 1996; Lautrou, 1982; Merz et al., 1991; Peretz et al., 1996; Scott and Symons, 1961; Williams et al., 1989). Form can be viewed as a combination of size and shape. Shape refers to the structure independent from its orientation, relation to reference planes, and dimension (or size) (Lestrel, 1989). Exhaustive quantitative information exists about dental size only, in particular about linear measurements of both crown and root (Abe et al., 1996; Anderson et al., 1977; Ash, 1993; Dalidjan et al., 1995; DuBrul, 1986; Hattab et al., 1996; Lautrou, 1982; Merz et al., 1991; Peretz et al., 1996; Wood et al., 1991). Indeed, the conventional metric measurements commonly used in biological studies provide quantitative information about size

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but do not completely separate the effects of size and shape on form and form modifications (Ferrario et al., 1995a; Lestrel, 1989; Lestrel et al., 1977; Lowe et al., 1994).

A recent electromyographic and kinesiographic investigation found significant sexual dimorphism in some functional characteristics of human chewing and hypothesized that morphological dentoskeletal variations could be somehow linked to such sex-related behavior (Gerstner and Parekh, 1997). As far as size is concerned, the crowns of human teeth have been reported to be larger in males than in females (e.g., Abe et al., 1996; Hattab et al., 1996; Merz et al., 1991; Wood et al., 1991), even if this sexual dimorphism is low. Conversely, no data exist about the sex-related characteristics of crown shape.

Dental shape is usually analyzed from a qualitative point of view (e.g., presence of nonmetric characteristics) or using angles or ratios between different measurements (Kelley, 1995; Kitagawa et al., 1996; Peretz et al., 1996). Indeed, angles and ratios provide not only just a crude estimate of shape, but also ratios are not really size-independent (Albrecht et al., 1993; Kelley, 1995) and do not allow separate quantification of the size and shape components of the biological form. For instance, shape descriptions of the occlusal surface have analyzed the relative arrangement of dental cusps (Peretz et al., 1996), but no quantitative analyses of the boundary of the occlusal surface exist. The size and shape of the occlusal surface and of the dental equator (maximum circumference of the dental crown) as well as their relative positions have several biomechanical, anthropological, and clinical implications, and a detailed description of their quantitative aspects could be of interest in various basic and applied biological fields, not least for the assessment of sexual dimorphism. For instance, the maxillary and mandibular occlusal surfaces come into contact during mandibular movements and in particular during chewing, and they are the areas where the masticatory forces discharge (Ash, 1993). The mesial and distal parts of the maximum circumference of the dental crown connect the teeth of the arch, producing a single working unit. In prosthetic reconstructions, the position, the dimension, and the shape of the dental equator should be taken into account to avoid deformations while taking impressions of dental arches. Anthropological descriptions of the form of dental surfaces are of current use for the determination of individuals and species (Wood et al., 1991). Among all human teeth, those with the least morphological irregularities (Ash, 1993) and with the largest biomechanical implications are the first molars, the largest elements within the arch and the first permanent teeth present in the mouth.

MATHEMATICAL APPROACHES TO OUTLINES OF CIRCUMFERENCE, SIZE, AND SHAPE

A quantitative analysis of the outlines of the occlusal and dental maximum circumference should be performed with mathematical methods that allow both a global assessment of the entire outline and a separation of the size and shape components of the form. Each outline can be considered a continuous function that can be decomposed into a sum of waves at different frequency and analyzed by curve-fitting procedures. One of these mathematical methods, devised by the French Jean Baptiste Fourier (1768–1830) and published at the beginning of the last century, separates complex forms into a series of cosine and sine functions of increasing frequency and varying amplitude (Johnson et al., 1985, 1992). The sum of these components gives a mathematical (and graphical) reconstruction of the original outline. The Fourier series are infinite, and the amplitude of each harmonic quantifies its relative contribution to the description of the form. Earlier members of Fourier series express lower frequency aspects of shape and are more important because most of the shape information is low frequency. High frequency harmonics therefore are numerically small (i.e., they have small coefficients) and express high frequency details that describe finer aspects of the form or minor differences in shape variation between groups. The addition of high frequency components allows a finer description of the form, but it is also influenced by measurement errors (i.e., by the noise of the system used to detect and digitalize the analyzed

outline). Fourier series are thus truncated after a finite number of harmonics, whose number depends on the irregularity of the original outline (Johnson et al., 1985, 1988, 1992; Lestrel et al., 1977; Schmittbuhl et al., 1998). The method analyzes the global shape characteristics of the outline and controls for size differences, differing spatial orientations, and the dependency on reference planes. In particular, size could be a confounding factor in the analysis of shape changes, because its modifications are often of greater magnitude than those of the corresponding shape (Ferrario et al., 1996a; Lestrel and Kerr, 1993; Lestrel et al., 1977). Each shape is thus described by a series of numbers, which allows for a comparison of different shapes and gives an estimate of the difference.

The classic Fourier series are thus the sum of one sine and one cosine component for each harmonic (Lestrel et al., 1977; Lu, 1965). A more recent mathematical development, the elliptic Fourier analysis (Kuhl and Giardina, 1982), conversely uses four coefficients (two sines and two cosines) for each harmonic. This method, suitable for the quantitative description of closed forms, transforms the outline of the objects into the sum of a series of ellipses. According to Lestrel (1989), the classic Fourier method is not really coordinate-free because a centroid is needed and a starting point should be defined (Johnson et al., 1985; Lestrel et al., 1977). Conversely, the elliptic method would overcome such a limitation because the orientation is performed using the center of the first elliptic harmonic and its major axis (Ferrario et al., 1996a). Nevertheless, the classic method has the advantage that the coefficients of the harmonics can have a distinct geometric meaning: the numerical differences in the Fourier coefficients can be related to actual differences in the observed morphology (Ferrario et al., 1997c; Lestrel et al., 1977; Lu, 1965; Schmittbuhl et al., 1998). In each harmonic, the sine term measures asymmetry, while the cosine term measures symmetry in respect to the x-axis (Johnson et al., 1985, 1988, 1992; Lestrel et al., 1977; Lu, 1965). The cosine coefficient of the 0-harmonic measures the size of the form and therefore should be constant or similar in all cases when all the outlines are standardized for size. In polar coordinates, the first harmonic is a circle shifted towards the region of major area, the second harmonic is a lemniscate, the third harmonic is a three-leaved figure, and each further harmonic adds one more leaf to the figure; thus each high frequency harmonic appears as a circle (Lu, 1965; Lestrel et al., 1977).

Both classic Fourier series and elliptic Fourier analysis have already been successfully used for quantitative studies of biological forms of man and mammals in several fields: neurology, dentistry, osteology, hematology, oncology (Casanova et al., 1990; Diaz et al., 1989; Ferrario et al., 1992, 1994b, 1995a, 1996a,c, 1997b,c; Halazonetis et al., 1991; Johnson et al., 1985, 1988, 1992; Lestrel, 1989; Lestrel and Kerr, 1993; Lestrel et al., 1977; Lowe et al., 1994; Lu, 1965; Schmittbuhl et al., 1998; Shen et al., 1994). For instance, the method allowed the quantification of shape changes during growth (Ferrario et al., 1996c, 1997c; Halazonetis et al., 1991; Johnson et al., 1988; Lu, 1965), aging (Ferrario et al., 1994b), and therapy (Lestrel and Kerr, 1993), the differences between species (Johnson et al., 1985; Lestrel et al., 1977; Schmittbuhl et al., 1998) and sexes (Ferrario et al., 1992, 1995a, 1997b; Halazonetis et al., 1991; Lu, 1995), and the effect of disease on human tissues and organs (Casanova et al., 1990; Shen et al., 1994) independently from more or less conspicuous size variations.

PURPOSE OF THIS STUDY

The classic Fourier series thus appear to be a sensible method for the assessment of morphological variations in human teeth, where the occlusal and maximum circumference of the first permanent molars, apart from pure morphological interest, also have several biomechanical implications. Numerical differences in selected harmonics may also be associated with geometrical characteristics of the outline. In this investigation, we attempt to analyze and describe the intrinsic (i.e., size-independent) morphologic characteristics of the outline of the human molar occlusal surface and maximum circumference. Fourier analysis (Ferrario et al., 1995a; Lestrel et al., 1977; Lu, 1965) will be used to quantify the shape of normal, healthy

first permanent molars and to assess the effect of sex. Size variations obtained from the mathematical reconstruction of dental outline will be also separately quantified.

MATERIALS AND METHODS

Sample

One hundred and thirty-one white Caucasian (Northern Italians) children (69 boys and 62 girls aged 8–13 years, mean 10 years) were selected from a group of 341 children attending a primary school where a mixed longitudinal and cross-sectional growth study is currently being performed. Selection of the children was performed according to the following criteria: no current orthodontic treatment; no temporomandibular or craniocervical disorders; and at least one completely erupted first permanent molar without occlusal wear, decays, conservative or prosthetic restorations. Moreover, no relatives were analyzed.

The parents of the children were previously informed about all test procedures and gave their consent to the investigation. Consent was also obtained from all the children.

The dental arches were reproduced from silicone double impressions (Provil; Bayer Dental, Leverkusen, Germany). The areas of each impression related to the first molars were isolated using Corning wax (Corning's Waxes; Corning Rubber Co., Brooklyn, NY), and the molars were cast in dental stone for fixed prosthodontic procedures (volume expansion 0.08%) (Kimberlit; Protechno, Sant Cugat, Spain). The occlusal surface of all molars was set horizontal using the Orthobox (Jolly Dental, Bologna, Italy), and a base was made. For the mandibular molars, the base of the tooth was set parallel to a plane touching the mesiolingual, distolingual, distobuccal, and mesiobuccal cusps, while for the maxillary molars the base of the tooth was set parallel to a plane touching the mesiobuccal, distobuccal, and mesiolingual cusps.

This procedure obtained data from 468 first permanent molars: 121 (57 female, 64 male) right and 120 (56 female, 64 male) left maxillary molars and 114 (56 female, 58 male) left and 113 (54 female, 59 male) right mandibular molars. Not all children had all four molars available for the analysis; therefore, each tooth was considered independently. A first qualitative analysis counted the occurrence of Carabelli's tubercle in the upper molars and of four, five, or six cusps in the lower molars separately for each sex.

Fig. 1. Occlusal view of the right mandibular first permanent molar (tooth 46) of a boy. The outline of the occlusal surface (marginal and cuspal ridges) is identified. The equatorial outline corresponds to the outer profile of the tooth.

A further selection was then made, and all teeth with incomplete eruption (101 teeth) and wear facets (24 teeth) that were not apparent at the first clinical examination or with technical imperfections (bubbles or deteriorations of the cast, 84 teeth) were discarded. The residual 259 teeth were further quantitatively analyzed (size and shape analysis).

Digitization and mathematical reconstruction of teeth

Digitization. A TV camera photographed each tooth (Fig. 1), and a single operator first traced and then digitized the outlines of the occlusal surface (marginal and cuspal ridges) and of the maximum circumference (equator) using an image analyzer (IBAS; Kontron, Munich, Germany). The system was calibrated on the focus plane of the occlusal surface before each acquisition, thus providing real metric data. Each outline was automatically digitized by the image analyzer (Ferrario et al., 1996c). About 120 points for each occlusal outline and 130

points for each equatorial outline were obtained. The x,y-coordinates of each point of the occlusal and equatorial outlines were used for the Fourier analysis of the dental form.

Fourier analysis of dental outlines. The method has been described in detail elsewhere (Ferrario et al., 1995a). All subsequent analyses were performed using algorithms created by one of the authors (V.F.F.) on a Bravo MS-T P/90 computer (AST, Irvine, CA) separately for each tooth and occlusal/equatorial outline.

Centroid calculation The geometric centroid of the area within the boundary was calculated and set as the origin of Cartesian axes. Calculation of the centroid was performed with the following procedure. First an approximated "point" centroid was computed from the Cartesian x- and y-coordinates of the boundary points. Consequently, a series of triangles with vertices in the point centroid and in each pair of contiguous points was mathematically constructed and the centroid of each triangle obtained. The final geometric ("area") centroid was averaged between the Cartesian coordinates of the triangle centroids weighed with the area of the relevant triangle. A detailed description of the method can be found in Ferrario et al. (1995a). The areas of each occlusal and equatorial outline, as well as their ratio, were computed and used separately from the shape analysis. Also, the respective positions of the occlusal and equatorial centroids were calculated.

Normalization The form was then normalized with respect to its orientation relative to the coordinate axes and to its size. The orientation of the lower teeth was normalized by setting the lingual groove horizontal (parallel to the x-axis), while the orientation of the upper teeth was normalized by setting the buccal groove horizontal (parallel to the x-axis). After the calculation of dental occlusal and equatorial areas, size normalization was performed mathematically with expansions or contractions of the area that did not modify the shape. For all teeth, the equatorial area was set equal to an arbitrary value

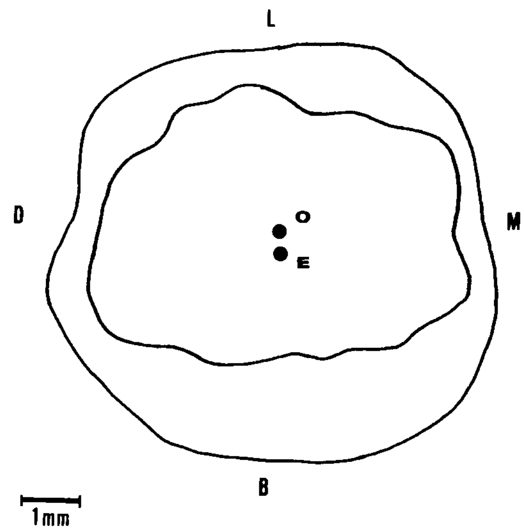

Fig. 2. Fourier reconstruction of the occlusal (inner trace) and equatorial (outer trace) outlines of the same molar shown in Fig. 1. The geometric centroids of the two outlines (E, equatorial; O, occlusal) are indicated. The relevant Fourier coefficients are listed in Table 1; B, buccal; L, lingual; M, mesial; D, distal sides of the molar.

of 7,000 mm^2, and the occlusal area was set equal to 5200 mm^2.

Curve fitting and harmonic analysis From the centroid (origin of axes) a mathematical vector was rotated counterclockwise for 360° with a 1° step to intersect the contour of the form, and 360 segments were produced. The length of each vector (modulus, or distance from the centroid to one of the 360 points of the contour) together with the relevant vector angle relative to the horizontal was used to generate a curve in a Cartesian coordinate system: y, vector modulus; x, angle (from 1° to 360°). A Fourier analysis of the curve was then performed, with period $\tau = 360$ (Lu, 1965). A previous investigation (Ferrario et al., 1995b) has already shown that the first 15 sine and cosine components in the Fourier series explained the analyzed dental shapes almost completely. Series were thus truncated at the fifteenth harmonic (Fig. 2; Table 1).

The Fourier series expansion is defined as

$$y = \frac{a_0}{2} + \sum_{m=1}^{15} [a_m \cdot \cos(\Theta \cdot m \cdot x) + b_m \cdot \sin(\Theta \cdot m \cdot x)]$$

TABLE 1. The first 15 Fourier coefficients of occlusal and equatorial outlines of tooth 46 shown in Figures 1, 2

	Occlusal		Equatorial	
H[1]	Cosine	Sine	Cosine	Sine
0	34.31135		47.15941	
1	−0.12522	0.31692	0.01576	0.01810
2	6.90497	0.25968	1.89716	−0.37632
3	0.91639	−1.53958	−0.93046	−0.47726
4	0.16759	1.11782	−1.50165	−0.17820
5	−0.93146	−0.34276	−0.12124	−0.64531
6	−0.90280	−0.10549	0.15683	0.48511
7	−0.02282	−0.32409	−0.10197	−0.60979
8	−0.95500	−0.24440	−0.02633	0.26204
9	−0.19277	0.22757	−0.02045	−0.34822
10	−0.47530	−0.50683	0.23658	0.01387
11	−0.13799	−0.25979	−0.00065	−0.10430
12	−0.18891	0.08658	0.03089	−0.01535
13	−0.26702	−0.07669	−0.09913	−0.12031
14	0.12081	0.40150	0.00816	0.04956
15	0.02399	0.24677	−0.04315	−0.05166

[1] Harmonics.

where

m = harmonic

a_m = cosine coefficient of the m harmonic

b_m = sine coefficient of the m harmonic

$\theta = 2\pi/\tau$, with τ normalized profile length.

Goodness of fit The goodness of fit for each individual curve (i.e., the agreement between the observed/digitized values and the values estimated according to the Fourier series) was calculated as proposed by Lu (1965) from the variances of the estimated and observed data.

Symmetry analysis. The percentage contribution of the sine (asymmetry) and cosine (symmetry) coefficients of the Fourier series to the mathematical description of each dental occlusal and equatorial outline was also computed. The buccolingual axis of each tooth was chosen as the reference axis for the calculation of symmetry. Obviously, the choice of a different axis would have modified the results, but the buccal and lingual grooves were well-defined morphological characteristics identifiable in all analyzed teeth. Calculations were performed as follows. All sine and cosine coefficients from the first to the fifteenth harmonics inclusive were squared and summed; the percentage ratio between the sum of the squared cosine coefficients and the total sum gave the symmetry contribution, while the percentage ratio between the sum of the squared sine coefficients and the total sum gave the asymmetry contribution (Ferrario et al., 1997b). Apart from rounding due to approximation, the sum of the two components should be 100%. The coefficients of 0-order were excluded from the calculation (Johnson et al., 1985) because the cosine coefficient is constant after size standardization, and the sine coefficient is always zero (Lestrel et al., 1977).

Size and shape comparisons. The difference between pairs of dental forms (contralateral teeth within an arch, homologous teeth between sexes) was computed separately for the size and shape components; size difference was appreciated from the original (before size standardization) area ratio, while shape difference was calculated from the size-, orientation-, and rotation-normalized Fourier reconstruction of the outlines (Ferrario et al., 1995a,b) as the differential percentage area (difference between the standardized areas to be compared as a percent of the sum of the same areas) of each pair of teeth. Before calculations, left teeth were mirrored at 180° on right teeth.

Statistical calculations and comparisons. For both occlusal and equatorial outlines, descriptive statistics (mean and standard deviation) for each coefficient as well as for the percentage contribution of sine and cosine coefficients were calculated within sex and tooth (left and right upper and lower first molars) from the coefficients of each dental outline. The significance of the coefficients within group was evaluated with a factorial analysis of variance (Lu, 1965). To allow us to find possible significant subgroups of different homologous teeth, Fourier coefficients also entered a cluster analysis (Anderberg, 1973).

Descriptive statistics were also computed for the occlusal and equatorial areas, for their ratio and for the reciprocal positions of the occlusal and equatorial centroids. The normality of all distributions was examined by inspecting the skewness and kurtosis of each data set.

Sex or side differences in the occurrence of morphologic characteristics were tested by

Chi-square tests, while differences in dental areas or equatorial-to-occlusal area ratios were tested by analyses of variance. In all tests, significance was set at the 5% level ($P \leq 0.05$).

Error of method

The method error has already been quantified and reported in abstract form elsewhere (Ferrario et al., 1995b). The right mandibular first molars of five adult subjects (one woman, four men) with excellent dentition were reproduced, digitized, and mathematically analyzed with the method previously described. Adult subjects were used for the assessment of the method error because it was not possible to take more than one impression of the same child's dental arches. In all cases, the molars had five cusps and were sound (no restorations or occlusal wear) and perfectly reproduced (no bubbles).

Four possible sources of error were tested: repeatability of silicone impressions, casting, dental orientation under the TV camera, and digitization. For silicone impressions, the lower dental arches of four men were reproduced twice, the right first molars analyzed, and the Fourier coefficients of their occlusal and equatorial outlines compared. For casting, the five original impressions were cast twice and the two sets of right mandibular first molars analyzed. For dental orientation under the TV camera, the same right mandibular first molar was repositioned and digitized five times. For tracing and digitization, the same operator traced and digitized the same set of five molars three times.

For each repeated measurement, the size difference was appreciated from the original (before size standardization) area ratio, while the shape difference between the size-, orientation-, and rotation-normalized Fourier reconstructions of the outlines (Ferrario et al., 1995a,b) was computed as the differential percentage area (difference between the standardized areas to be compared as a percent of the sum of the same areas) of each couple of teeth.

Other negligible errors could derive from the mathematical procedure, in particular from the approximation algorithms.

RESULTS

Error of method

All the teeth used for this procedure were well reconstructed by Fourier series. The mathematical comparison of the repeated analyses of the same sets of teeth found no differences in size for both the equatorial and occlusal outlines, with area ratios ranging between 0.98 and 1.01, while some shape variations were found, as reported in Table 2. In particular, a second casting of the same impressions produced teeth with a somewhat different shape (error test 2). All the other tests yielded smaller errors. In all cases, equatorial shape repeatability was larger than occlusal shape repeatability.

TABLE 2. Method error: Shape differences between the size-, orientation-, and rotation-normalized Fourier reconstructed outlines

Error test[1]	Subject tooth[2]	% shape difference	
		Occlusal	Equatorial
1	m18–46	1.96	0.86
	m50–46	1.77	0.82
	m51–46	2.07	0.94
	m52–46	1.88	0.90
Mean		1.92	0.88
2	f50–46	6.29	1.05
	m18–46	5.81	1.37
	m50–46	7.84	1.02
	m51–46	4.70	2.01
	m52–46	3.70	1.96
Mean		5.67	1.48
3	m50–46a	1.68	0.99
	m50–46b	1.30	1.05
	m50–46c	1.20	1.00
	m50–46d	1.43	0.78
Mean		1.40	0.96
4	f50–46	0.88	1.01
	m18–46	1.29	0.75
	m50–46	1.37	0.79
	m51–46	1.10	0.76
	m52–46	2.77	1.44
Mean		1.48	0.95

[1] Error test: repeatability of silicone impressions (1), casting (2), dental orientation under the TV camera (3), and tracing and digitization (4).
[2] Teeth are coded with the WHO notation system: first digit: 1, maxillary right; 2, maxillary left; 3, mandibular left; 4, mandibular right; second digit: tooth number (in the present investigation, 6, first molar).

Qualitative observations

In this first qualitative analysis of molar form, the teeth with incomplete eruption and wear facets that were not apparent at the first clinical examination as well as those with slight technical imperfections were analyzed (468 teeth). In the right hemi-arch, Carabelli's tubercle was observed in 20 upper female molars (35.09% of 57 teeth) and in 26 male homologous teeth (40.62%). In the left hemi-arch, it was observed in 17 female (30.36%) and 25 male (39.06%) first permanent maxillary molars. No sex or side

TABLE 3. Descriptive statistics of the occlusal and equatorial areas of the first permanent molars[1]

		Equatorial area		Occlusal area		E/O area ratio	
Tooth[2]	N	Mean	S.D.	Mean	S.D.	Mean	S.D.
Girls							
16	40	89.10	7.73	52.51	5.37	1.703	0.124
26	31	92.10	7.04	52.99	4.68	1.743	0.123
36	30	87.66	10.73	51.21	7.82	1.728	0.132
46	30	89.84	12.52	52.20	8.73	1.731	0.127
Boys							
16	31	94.10	8.98	53.97	5.47	1.749	0.133
26	31	96.27	9.41	55.02	5.42	1.753	0.119
36	30	93.40	9.35	54.19	8.84	1.742	0.160
46	30	95.01	7.90	54.37	5.95	1.755	0.125

[1] E/O area ratio, equatorial to occlusal area ratio; N, number of teeth; S.D., standard deviation; Unit: areas, mm^2; area ratios, mm^2/mm^2.
[2] Teeth are coded with the WHO notation system: first digit: 1, maxillary right; 2, maxillary left; 3, mandibular left; 4, mandibular right; second digit: tooth number (in the present investigation, 6, first molar).

differences were found in the occurrence of Carabelli's tubercle (Chi-square test for sex within side, right side $\chi^2 = 0.192$, left side $\chi^2 = 0.649$; for side within sex, males $\chi^2 = 0$, females $\chi^2 = 0.112$; 1 degree of freedom for all comparisons; $P > 0.05$ in all cases).

Most of the first permanent mandibular molars had five cusps: 41 in the female left (74.55%) and 37 in the female right (71.15%) side and 49 in the male left (85.96%) and 48 in the male right (84.21%) side. No sex or side differences were found in the occurrence of five vs. four cusps (Chi-square test for sex within side, right side $\chi^2 = 1.192$, left side $\chi^2 = 1.645$; for side within sex, males $\chi^2 = 0$, females $\chi^2 = 0.031$; 1 degree of freedom for all comparisons; $P > 0.05$ in all cases). Six cusps were observed only in two first permanent mandibular left molars (one boy, one girl), and in four right homologous teeth (two boys, two girls). The four and six cusp molars were not further analyzed.

Fourier reconstruction of occlusal and equatorial outlines

Only 259 perfectly sound first permanent molars without technical defects were further quantitatively analyzed (Table 3). In all teeth, the Fourier series well reconstructed dental occlusal and equatorial shape, with coefficients of agreement higher than 0.91 when the series were truncated at the fifteenth harmonic. According to Lu (1965), this coefficient should be 0.6 at least, thus indicating a formal correlation of about 0.9 between the observed and estimated data, and it should be 1 for a perfect fit (coincidence between the observed and estimated data).

Size analysis. Both equatorial and occlusal molar areas were on average larger in male than in female homologous teeth (Table 3). The difference was statistically significant only for the equatorial areas (analysis of variance boys vs. girls: lower arch, F = 3.185, 3,116 degrees of freedom, $P \leq 0.05$; upper arch, F = 4.774, 3,129 degrees of freedom, $P \leq 0.01$). The mean ratios between equatorial and occlusal dental areas were similar in both arches and in both sides and slightly larger in boys than in girls but without significant differences (analysis of variance boys vs. girls: upper arch, F = 1.264, 3,129 degrees of freedom, $P > 0.05$; lower arch, F = 0.241, 3,116 degrees of freedom, $P > 0.05$).

Shape analysis. Mean occlusal and equatorial shapes were mathematically computed within sex and tooth. A large within-group variability was found, and when the significance of the Fourier coefficients within sex and tooth was evaluated with a factorial analysis of variance (Lu, 1965), only about half of the equatorial and occlusal sine and cosine coefficients were significant at the 5% probability level. Variability was particularly high in the mandibular teeth. Overall, the significance was larger for the even order harmonics. In particular, in the maxillary molars the second, fourth, and sixth/eighth order harmonic were significant in all cases. In the mandibular molars, the second

TABLE 4. Percentage contribution of the sine and cosine components in the mathematical reconstruction of the occlusal and equatorial outlines of the first permanent molars[1]

		Equatorial			Occlusal		
		Mean			Mean		
Tooth[2]	N	Cosine	Sine	S.D.	Cosine	Sine	S.D.
Girls							
16	40	33.95	66.05	18.59	74.14	25.86	11.59
26	31	36.19	63.81	17.90	79.05	20.95	12.12
36	23	63.38	36.62	13.82	88.46	11.54	5.69
46	21	65.31	34.69	14.70	90.67	9.33	4.44
Boys							
16	31	33.65	66.35	17.24	79.24	20.76	11.42
26	31	39.11	60.89	17.70	81.57	18.43	13.21
36	24	64.17	35.83	15.13	86.77	13.23	7.07
46	27	65.50	34.50	15.68	85.04	14.96	10.10

[1] N, number of teeth; S.D., standard deviation (the standard deviation is equal for both sine and cosine components).
[2] Teeth are coded with the WHO notation system: first digit: 1, maxillary right; 2, maxillary left; 3, mandibular left; 4, mandibular right; second digit: tooth number (in the present investigation, 6, first molar).

order harmonic was significant in all equatorial outlines and in all cosine coefficients of the occlusal outlines. The lack of widespread significance of coefficients prevented the correlation of morphological characteristics of the outlines with the numerical values of Fourier coefficients. The cluster analysis performed with Fourier coefficients failed to find significant subgroups of different homologous teeth; the larger clusters comprised only five teeth.

For each examined molar, the relative contribution of the asymmetry (sine) and symmetry (cosine) components of the Fourier series to the description of the dental outlines was computed (Table 4). In all teeth, on average cosines explained more than 70% of the shape of the occlusal contour, but the percentage cosine contribution was larger in the mandibular (between 85% and 92%) than in the maxillary teeth (between 74% and 82%), thus reflecting the higher degree of symmetry relative to the buccolingual axis of the lower molar outlines. The different contributions of sine and cosine components were more evident for the equatorial outlines; in the mandibular arch, cosines explained on average about 65% and in the maxillary arch 36% of the dental shape. Moreover, the within-sample variability for the relative sine/cosine contribution to the shape was larger for the equatorial outlines than for the occlusal outlines.

The geometric centroids of the occlusal and equatorial outlines were about coincident in the mesiodistal direction in all teeth, while differences in the buccolingual direction were found in all cases. Mean differences were larger in the mandibular than in the maxillary arch. In the upper arch, the occlusal outline was shifted in the buccal direction relative to the equatorial outline (on average 0.18 mm). Conversely, in the lower arch the occlusal outline was shifted in the lingual direction relative to the equatorial outline (on average 0.84 mm).

The mean Fourier coefficients, equatorial-to-occlusal area ratios, and relative positions of the geometric centroids were used to mathematically reconstruct mean dental shapes (Figs. 3 and 4). For each mean tooth, the typical dental occlusal shape was maintained (i.e., the upper molars were diamond-shaped or rhomboidal, while the lower molars were about quadrilateral) but this shape was extremely simplified and quite schematic. The equatorial shapes were similar to the corresponding occlusal shapes.

The outlines of left and right homologous molars were compared within sex and arch by computing the size (right:left area ratio) and shape difference (between the size-, orientation-, and rotation-normalized mean Fourier reconstructed outlines). Both equatorial and occlusal outlines of contralateral teeth within sex and arch were similar, without size and shape differences; area ratios were between 0.97 and 1 mm^2/mm^2, and shape differences were between 0.75% and 3%. Similarly, no sex differences in shape were found in the comparison of ho-

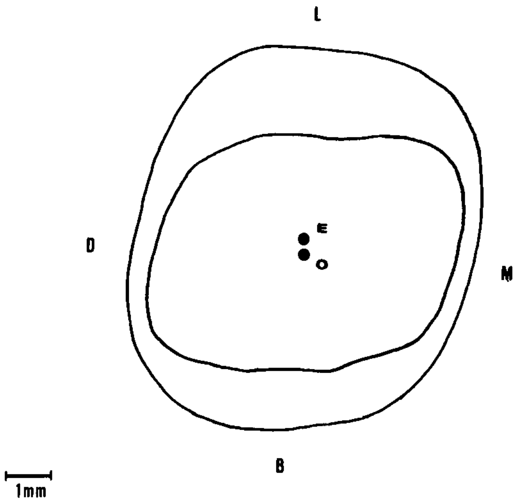

Fig. 3. Fourier reconstruction of the mean occlusal (inner trace) and equatorial (outer trace) outlines of the right maxillary first permanent molar in boys. The geometric centroids of the two outlines (E, equatorial; O, occlusal) are indicated; B, buccal; L, lingual; M, mesial; D, distal sides of the molar.

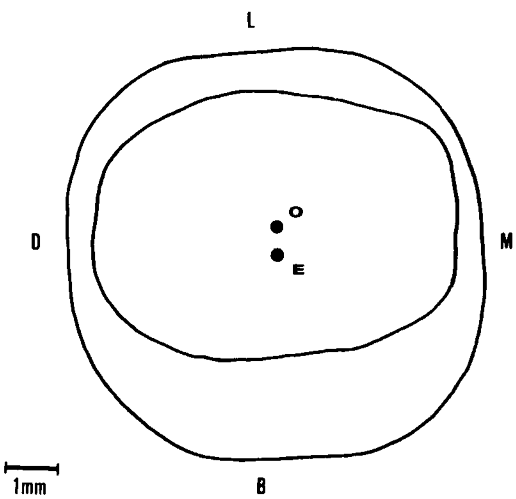

Fig. 4. Fourier reconstruction of the mean occlusal (inner trace) and equatorial (outer trace) outlines of the right mandibular first permanent molar in boys. The geometric centroids of the two outlines (E, equatorial; O, occlusal) are indicated; B, buccal; L, lingual; M, mesial; D, distal sides of the molar.

mologous teeth (shape differences between 0.67% and 2.33%).

DISCUSSION

In the present investigation, the normal form (size and shape characteristics) of the occlusal and equatorial outline of the human first permanent molars was quantified. Molar form was analyzed in the selected teeth of a group of living children, and all teeth were free from wear facets, decays, and conservative or prosthetic restorations. The teeth were reproduced with the conventional procedures used in prosthetic dentistry, and all elements with technical imperfections (bubbles or deteriorations of the cast) were eliminated. A second casting of the original impressions was not possible, because this procedure generated teeth with large shape differences in their occlusal outlines (Table 2, error test 2). All the other technical factors contributing to shape variations appeared to be negligible as assessed by the comparison between the relevant standardized Fourier reconstructions. Indeed, the number of teeth analyzed for the assessment of the method error was limited, and a more formal statistical quantification of the error was not possible. It has to be mentioned that the four sources of error analyzed in the present protocol were not completely independent; the second impression of the same tooth needed to be traced, orientated, and digitized independently from the first one, and the error source 1 (error test 1, Table 2) thus comprised also a contribution from sources 3 and 4. The same applied to the error of a second casting of the same impression (error test 2 in Table 2). Nevertheless, while the error of a second impression (occlusal outline) was on average only 1.5 times larger than the tracing, digitization, and orientation errors, the error of the second casting (occlusal outline) was on average three to four times larger than any of the other errors.

Ethical considerations prevented a second silicone impression in the same children (indeed, the procedure is not dangerous, but it could be troublesome, and we preferred to avoid its repetition in healthy, non-patient children).

Dental size was measured from occlusal (between the marginal and cuspal ridges) and equatorial (maximum dental circumference) dental areas. Previous studies have usually measured dental mesiodistal and buccolingual diameters or intercusp distances (Abe et al., 1996; Anderson et al.,

1977; Ash, 1993; Dalidjan et al., 1995; DuBrul, 1986; Hattab et al., 1996; Lautrou, 1982; Merz et al., 1991; Peretz et al., 1996; Scott and Symons, 1961; Wood et al., 1991). Present data on the occlusal area (Table 3) are well comparable with literature findings: for instance, Lautrou (1982), DuBrul (1986), Ash (1993), and Otuyemi and Noar (1996) report mean sex-independent crown diameters that could be used to approximate the occlusal surface with an ellipse. Considering that the internal occlusal surface (between the marginal and cuspal ridges) occupies about 50–60% of the total occlusal surface (comprised between the buccolingual and mesiodistal diameters) (Lautrou, 1982), estimated areas between 43 and 56 mm^2 (maxillary first permanent molar) and between 44 and 56 mm^2 (mandibular first permanent molar) could be obtained.

Significant sexual dimorphism in dental size was found only for the equatorial areas, the larger male occlusal areas not being significantly different from the homologous female ones. Significant sex differences in molar crown diameters were found in healthy Japanese adults (Abe et al., 1996), in healthy Jordanian adolescents (Hattab et al., 1996), and in Bantu-speaking Negroes (Wood et al., 1991). Sex differences in several dental measurements have also been reported in both white and black Americans (Merz et al., 1991). Conversely, intercusp distances in both normal children and children with Down's syndrome showed no sex differences (Peretz et al., 1996). Mean male:female area ratios found in the present investigation were smaller than similar ratios computed from the data reported by Abe et al. (1996): 1.033 vs. 1.099 in the upper arch and 1.05 vs. 1.115 in the lower arch. The contrasting literature findings could be explained by race- or ethnic-specific differences in sex-related dental size as well as by the different method used for the measurements. At any rate, this sexual dimorphism has been found to be low (Dalidjan et al., 1995) and of minor importance when compared to taxa differences (Wood et al., 1991). Larger crown dimensions in men could be related to longer dental development before eruption (Ash, 1993; Hattab et al., 1996), hormonal and sex-chromosome influences (Hattab et al., 1996), and larger body weight (Anderson et al., 1977). No literature data were found for equatorial measurements.

Values reported in the present study should be taken with some caution because the maximum diameter does not belong to a single plane along all dental facets. Moreover, measurements were estimated in the same TV focus plane used for the occlusal area. Indeed, little differences in the z-direction (focus depth) exist between the occlusal surface and the maximum dental diameter, but they were not calculated, and data were not corrected for such different magnifications.

Dental shape was assessed by both a macroscopic qualitative analysis (occurrence of Carabelli's tubercle in the upper molars and number of cusps in the lower molars) and by a quantitative Fourier reconstruction of the dental outlines. In both cases, no sex differences were found. The lack of significant sexual dimorphism in the shape of the occlusal and equatorial outlines of the first permanent molars can be associated with recent findings of no sex-related differences in the three-dimensional curvature of dental arches (Ferrario et al., 1997a) and in the soft-tissue facial shape of young adult individuals (Ferrario et al., 1994a, 1995a, 1996b). Indeed, the morphological basis for the sex-related functional differences in the stomatognathic apparatus (Gerstner and Parekh, 1997) might be just a size discrepancy, as already hypothesized by some investigators (Peck et al., 1992).

No differences in the size and shape of the occlusal and equatorial outline of left and right homologous teeth were found (i.e., the mean dental forms within sex and arch were symmetric). Obviously, this finding applies only to the mean outlines, because no assessment of the asymmetry within single subjects was made.

Literature data on the occurrence of Carabelli's tubercle in the upper molars are very disparate, and percentages between 10 and 15% (DuBrul, 1986) and 60% (Scott and Symons, 1961) have been reported. The present finding of 30–40% seemed therefore well within the large variability of the previous observations. Moreover, a racial difference in this nonmetric dental characteristic seems

to exist, it being more frequent in Caucasoid teeth (Williams et al., 1989). DuBrul did not report the ethnic composition of his sample, while the findings by Scott and Symons (1961) referred to white Americans.

Conversely, the percentage of four cusp maxillary molars found in the current study (between 15 and 30%) was larger than any other literature report; this characteristic was noted as "unusual" (Scott and Symons, 1961) and as "occasional" (Ash, 1993) or estimated to be present in only 5–6% of normal first permanent molars (DuBrul, 1986).

Fourier analysis allowed a quantitative comparison between male and female dental shapes as well as between left and right homologous teeth. In both cases, no differences larger than the method error (Table 2) were found. This last finding is difficult to compare with other investigations because no data about intrinsic dental shape exist. Previous reports analyzed dental indices and ratios (Kelley, 1995) or the relative arrangement of dental cusps (Peretz et al., 1996). Moreover, neither of those two investigations are directly relevant in the present contest; Peretz et al. (1996) analyzed the effect of a chromosomal alteration (trisomy 21) on human dental shape, and Kelley (1995) studied monkey teeth.

Whenever closed forms are considered, the classic Fourier series could be replaced by the elliptic Fourier series (Diaz et al., 1989; Ferrario et al., 1994b, 1996a,c; Lestrel, 1989; Lestrel and Kerr, 1993; Lowe et al., 1994). Unfortunately, in this case each harmonic is an ellipse (Diaz et al., 1989; Lestrel and Kerr, 1993), and their geometrical effect on the contour of complex shapes could be difficult to interpret. For instance, in the present application the sine and cosine terms gave information about dental symmetry (Table 4), an assessment that cannot be performed with elliptic descriptors. The method applied in this investigation is suitable for both open (Ferrario et al., 1992, 1997b; Halazonetis et al., 1991; Lestrel et al., 1977; Lu, 1965) and closed forms (Casanova et al., 1990; Ferrario et al., 1995a, 1997c; Halazonetis et al., 1991; Johnson et al., 1985, 1988, 1992; Schmittbuhl et al., 1998; Shen et al., 1994). Moreover, whenever groups of forms have to be compared, the analysis of variance for the comparison of Fourier coefficients (which satisfy the property of orthogonality and can thus be further analyzed with conventional statistics) has already been exemplified in detail by Lu (1965) and Lestrel et al. (1977). The geometric meaning of the coefficients of the harmonics could not be correlated to the actual outlines of the mean teeth because of the lack of a widespread significance. Tentatively, the larger number of significant even order harmonics could be explained by a general symmetric appearance of the dental outlines.

A similar procedure was employed by Halazonetis et al. (1991). Those authors graphically superimposed the mandibular outline on the Fourier waveforms plotted in cartesian x, y coordinates, and evaluated the approximated area of influence of each Fourier coefficient on a local basis. In the present analysis, the influence of the sine and cosine coefficients of each harmonic was evaluated on the global cephalometric outline.

The analysis was limited to the first fifteen harmonics only because higher degree waves had too small amplitudes. As already pointed out, harmonics with a small amplitude contribute relatively less to the total form (Johnson et al., 1992).

A limitation to the method used in the present study seemed to be its extreme sensitivity to local variations in dental shape. Indeed, there seems to be an appreciable random component in dental shape. While the major landmarks (those used for the mathematical alignment of teeth) maintained a relatively constant position (the overall form of the molar outline was preserved), the dental outline between them was variable. Therefore, when mean dental shapes within sex, arch, and side were plotted (Figs. 3, 4), the resultant outlines resembled extremely schematized normal teeth: small individual variations in the position of cusps, grooves and ridges were averaged, and the global dental outline was simplified and unnatural. This effect was the graphical result of the large within-group mathematical variability found in Fou-

rier coefficients, which prevented a widespread significance of the same coefficients.

The method used in the current investigation could thus supply information about the dental shape in both its entirety and local variations, and it could be usefully employed to compare single teeth to a standard. For example, it could quantify any asymmetry in shape between homologous teeth in a single individual, or it could be used to control for size variations in artificial (prosthetic) teeth.

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